Vector-Meson Production in Meson-Baryon Collisions

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A covariant general treatment of the amplitude for isoscalar vector-meson production in pseudoscalar meson-baryon collisions is presented. For the low-energy region, a model is devised by assuming the invariant amplitudes to obey a one-dimensional dispersion relation. The formalism is applied to the reaction pion +nucleon → ω meson+nucleon, for which the single-nucleon and the pion-nucleon cut are taken to dominate the direct channel, while the crossed channels are described by ρ-meson and nucleon-exchange processes. In the energy range of interest, the pion-nucleon cut is further approximated by the isospin $\frac{1}{2}$ isobars. In the multipole amplitudes of the resonances, for which a Breit-Wigner formula is assumed, only the lower ω angular momentum is exhibited. The differential and the total cross sections are thus expressed in terms of the coupling constants involved and the resonant partial widths, which are to be determined by comparing with the available experimental data. A satisfactory fit is obtained using only the Born terms as a preliminary step, with the following values for the coupling constants:

$$g_\pi^2/4\pi=14.4,\ g_{\omega\rho\pi}^2/4\pi=10.5,\ g_\omega^2/4\pi=1.0,\ g_{\rho c}^2/4\pi=0.4,\ g_{\rho m}^2/4\pi=0.4,\ \text{and}\ g_{\rho m}/g_{\rho c}=-1.0.$$

I. INTRODUCTION

DURING the past few years, experiments have revealed the existence of several unstable vector mesons. All these mesons—ρ, ω, K^*, φ being the best established ones—decay via strong interactions. Consequently, they have been considered to be resonances by some physicists, while others would prefer to regard them as being as fundamental as any other particles.

An S-matrix approach to the production of these unstable mesons encounters the difficulty that in such a theory the transitions are defined only between asymptotic states. A possibility to overcome this obstacle would be to treat the unstable particle as a resonance of the multiparticle final state. Such an approach was used by several authors.[1] However, this kind of program seems to be even more difficult to realize for the ω-meson ("3 π-resonance") production. Hence, we shall treat in this work the vector mesons as metastable particles, as far as their production is concerned. We shall explicitly consider the production of vector mesons having zero isotopic spin, which appear to have the smaller decay widths (<10 MeV), and for which our approach should be more suitable.

The interesting physical quantities we intend to learn by comparing our theoretical treatment with experimental data are the coupling constants of the vector mesons to nucleons. Several theoretical papers have already appeared,[2] in which information on these coupling constants is sought by calculating the nucleon-nucleon scattering amplitude with these mesons entering into intermediate states and comparing it with the experimental observables from proton-proton and neutron-proton scattering in the low-energy (<400-MeV) region. We feel that a theoretical analysis of the production process of vector mesons offers an independent and more direct way to the study of the vector-meson-nucleon interaction.

In this work we shall study the vector-meson production through the process pseudoscalar meson+baryon→ vector meson+baryon. The quantum numbers of the particles here suggest a comparison to the meson photoproduction process treated by Chew, Goldberger, Low, and Nambu.[3] In their classical paper which applies a dispersion theoretic approach, use is made of unitarity in order to relate the pion photoproduction amplitudes to pion-nucleon scattering phase shifts. Unfortunately, such a simple relation does not exist in our case. It is now much more complicated due to the fact that there are many competing open channels coupled to the process of interest. However, we shall assume that single-production channel treatment and a Cini-Fubini one-dimensional dispersion relation[4] are valid in the low-energy production region. There have been recently several theoretical articles in which pion photoproduction,[5] photoproduction of strange particles[6] and asso-

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[1] L. F. Cook and B. W. Lee, Phys. Rev. **127**, 283 and 297 (1962). J. S. Ball, W. R. Frazer, and M. Nauenberg, *ibid.* **128**, 478 (1962). P. Federbush, M. T. Grisaru, and M. Tausner, Ann. Phys. (N. Y.) **18**, 23 (1962). S. Mandelstam, J. E. Paton, R. F. Peierls, and A. Q. Sarker, *ibid.* **18**, 198 (1962).

[2] S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **28**, 991 (1962). A. Scotti and D. Y. Wong, Phys. Rev. Letters **10**, 142 (1963) and to be published. R. A. Bryan, C. R. Dismukes and W. Ramsay, Nucl. Phys. **45**, 353 (1963). W. Ramsay, Phys. Rev. **130**, 1552 (1963).

[3] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[4] M. Cini and S. Fubini, Ann. Phys. (N. Y.) **10**, 352 (1960).

[5] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963). Ph. Salin, *ibid.* **28**, 1294 (1963). D. S. Beder, *ibid.* (to be published).

[6] S. Hatsukade and H. J. Schnitzer, Phys. Rev. **128**, 468 (1962); **132**, 1301 (1963). M. Gourdin and J. Dufour, Nuovo Cimento **27**, 1410 (1963). T. K. Kuo, Phys. Rev. **129**, 2264 (1963); **130**, 1537 (1963).

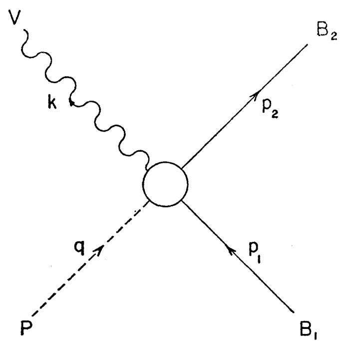

FIG. 1. Diagram for the production process.

ciated production by pions[7] were treated quite successfully along similar lines.

The analysis of a specific reaction, $\pi+N\to\omega+N$ is carried out in detail. Besides all the Born terms, the spectral functions will be approximated by the existing resonant states in the respective channels. A preliminary but suggestive numerical analysis is done and compared with the experimental result by Fields *et al.*[8] Speculations have been occasionally made[9] that production processes of some of the vector mesons occur mainly through a vector-meson (sometimes plus a pseudoscalar-meson) exchange. Although this exchange mechanism might be important for certain processes in a certain energy range, in our case of ω production, the single-particle exchange, which would be a ρ-meson exchange, is not supported by available experimental data.[8] We give arguments in Sec. III to explain why this simple picture is not adequate, and consequently we feel that a more complete theoretical attempt is needed.

In Sec. II we give the general formalism and kinematics of the production process. In Sec. III the model for pion production of the ω meson is presented. Section IV gives the various interaction amplitudes within our model. In Sec. V numerical results with only pole terms are given and compared with experimental data. In a future publication a more complete numerical analysis will be presented. A discussion is given in Sec. VI.

II. KINEMATICAL AND GENERAL CONSIDERATIONS

Let us consider the general reaction, pseudoscalar meson+spin $\frac{1}{2}$ baryon $\to$ vector meson+spin $\frac{1}{2}$ baryon. The energy-momentum four-vectors are denoted by q for the pseudoscalar meson of mass M_p, p_1 for the incoming baryon of mass M_1, k for the vector meson of mass M_v, and p_2 for the outgoing baryon of mass M_2 (see Fig. 1).

We use the familiar invariant variables:

$$\begin{aligned} s&=(q+p_1)^2=(k+p_2)^2\,,\\ t&=(k-q)^2=(p_2-p_1)^2\,,\\ u&=(p_1-k)^2=(p_2-q)^2\,, \end{aligned}\tag{2.1}$$

with

$$\begin{aligned} q+p_1&=k+p_2\,,\\ s+t+u&=M_1^2+M_2^2+M_p^2+M_v^2\,. \end{aligned}\tag{2.2}$$

The invariants s, t, u are the squares of the total center-of-mass energies in the s, t, and u channels defined as follows:

$$\begin{aligned} P+B_1&\to V+B_2\,, \quad s\text{ channel}\,,\\ P+\bar{V}&\to \bar{B}_1+B_2\,, \quad t\text{ channel}\,,\\ \bar{V}+B_1&\to \bar{P}+B_2\,, \quad u\text{ channel}\,. \end{aligned}\tag{2.3}$$

The T matrix for the production process is related to the S matrix by

$$S_{fi}=-\frac{i}{(2\pi)^2}\delta^{(4)}(q+p_1-k-p_2)\times\left(\frac{M_1M_2}{4q_0k_0p_{10}p_{20}}\right)^{1/2}\bar{u}(p_2)Tu(p_1)\,. \tag{2.4}$$

The most general covariant form of the T matrix can be obtained as outlined in Ref. 3. There are five independent four-vectors, ϵ, q, $P=\frac{1}{2}(p_1+p_2)$, k and γ, where ϵ is the polarization four-vector of the vector meson and γ is the Dirac spin operator.[10] By using the Dirac equation, the commutation properties of γ matrices and the Lorentz condition $\epsilon\cdot k=0$ which eliminates the scalar part, one finds that the T matrix can be written as a linear combination of six independent Lorentz-invariant operators.

$$T(s,t,u)=\sum_{i=1}^{6}A_i(s,t,u)O_i\,. \tag{2.5}$$

The decomposition of T is not unique. We choose the set of O_i such that A_i contain no kinematical singularities[11] and thus can be assumed to satisfy a Mandelstam representation. We have

$$\begin{aligned} O_1&=\gamma_5\gamma\cdot\epsilon\,, & O_4&=\gamma_5\gamma\cdot\epsilon\gamma\cdot k\,,\\ O_2&=2\gamma_5\epsilon\cdot P\,, & O_5&=2\gamma_5\gamma\cdot k\epsilon\cdot P\,,\\ O_3&=\gamma_5\epsilon\cdot q\,, & O_6&=\gamma_5\gamma\cdot k\epsilon\cdot q\,. \end{aligned}\tag{2.6}$$

The scalar functions A_i are matrices in isotopic spin

[7] M. Gourdin and M. Rimpault, Nuovo Cimento **20**, 1166 (1961); 24, 419 (1962). A. Kanazawa, Phys. Rev. **123**, 997 (1961). T. Tsuchida, T. Sakuma, and S. Furui, Progr. Theoret. Phys. (Kyoto) **26**, 1005 (1961).

[8] T. Fields, S. Orenstein, R. Kraemer, L. Madansky, M. Meer, A. Pevsner, C. Richardson, and T. Toohig, in *Proceedings of the Athens Topical Conference on Recently Discovered Resonant Particles, Athens, Ohio, 1963* (University of Ohio, Athens, Ohio, 1963), p. 185, and to be published. T. Fields (private communication).

[9] G. A. Smith, J. Schwartz, D. H. Miller, G. R. Kalbfleisch, R. W. Huff, O. I. Dahl, and G. Alexander, Phys. Rev. Letters **10**, 138 (1963). R. W. Huff, Phys. Rev. **133**, B1078 (1964). See, however, S. M. Flatté, R. W. Huff, D. O. Huwe, F. T. Solmitz, and M. L. Stevenson, Bull. Am. Phys. Soc. **8**, 603 (1963).

[10] We use the following definitions for the γ matrices: $\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$; $\gamma_0{}^*=\gamma_0$; $\gamma_k{}^*=-\gamma_k$, $k=1, 2, 3$; $\gamma_5=\gamma^0\gamma^1\gamma^2\gamma^3$. Asterisk denotes here Hermitian conjugation. The metric used is $g_{00}=-g_{11}=-g_{22}=-g_{33}=1$ and the units employed throughout are $\hbar=c=1$.

[11] J. S. Ball, Phys. Rev. **124**, 2014 (1961).

space. Their explicit representation varies according to the kind of particles involved in the reaction.

As stated in the introduction we shall assume the spectral functions to be dominated by resonances. Since these resonances have definite angular momentum and parity, it is more convenient to express A_i in terms of center of mass amplitudes which are then written as multipole expansions. In the c.m. system of the s channel, we define

$$q=(q_{0'}\mathbf{q}),\qquad k=(k_{0'}\mathbf{k}),$$
$$p_1=(E_{1'}-\mathbf{q}),\quad p_2=(E_{2'}-\mathbf{k}). \tag{2.7}$$

The three scalar variables are

$$s=W^2,$$
$$t=M_v^2+M_p^2-2k_0q_0+2kq\cos\theta, \tag{2.8}$$
$$u=M_1^2+M_v^2-2E_1k_0-2qk\cos\theta,$$

where W is the total c.m. energy and θ is the c.m. angle between the vector meson and the incident pseudoscalar meson. From now on we denote $k=|\mathbf{k}|$, $q=|\mathbf{q}|$. The momenta and energies are

$$q=\frac{\{[s-(M_1-M_p)^2][s-(M_1+M_p)^2]\}^{1/2}}{2W},$$
$$k=\frac{\{[s-(M_2-M_v)^2][s-(M_2+M_v)^2]\}^{1/2}}{2W}, \tag{2.9}$$
$$q_0=\frac{s-(M_1^2-M_p^2)}{2W},\quad k_0=\frac{s-(M_2^2-M_v^2)}{2W},$$
$$E_1=\frac{s+(M_1^2-M_p^2)}{2W},\quad E_2=\frac{s+(M_2^2-M_v^2)}{2W}.$$

The differential cross section for vector-meson production is

$$d\sigma/d\Omega=(k/q)|\chi_f{}^\dagger F\chi_i|^2, \tag{2.10}$$

where χ is a Pauli spinor. The F matrix is related to T matrix as follows:

$$\chi_f{}^\dagger F\chi_i=[(M_1M_2)^{1/2}/4\pi W]\bar{u}(p_2)Tu(p_1), \tag{2.11}$$

with

$$u(p)=\frac{\gamma\cdot p+M}{[2M(E+M)]^{1/2}}\begin{pmatrix}\chi\\0\end{pmatrix}.$$

The F matrix may be written as

$$F=i\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}F_1+\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}}{qk}F_2+\frac{i\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon}}{kq}F_3$$
$$+\frac{i\boldsymbol{\sigma}\cdot\mathbf{q}\mathbf{q}\cdot\boldsymbol{\varepsilon}}{q^2}F_4+\frac{i\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{k}\cdot\boldsymbol{\varepsilon}}{k^2}F_5+\frac{i\boldsymbol{\sigma}\cdot\mathbf{q}\mathbf{k}\cdot\boldsymbol{\varepsilon}}{qk}F_6. \tag{2.12}$$

The F matrix is further expanded into multipole transitions through projection operators[12]

$$F=\sum_{l=0}^{\infty}\left\{\left[-iM_l{}^+\boldsymbol{\varepsilon}\cdot\mathbf{l}_k-E^-{}_{l+1}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\varepsilon}\cdot\mathbf{k}\times\mathbf{l}_k}{qk}+iL^-{}_{l+1}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\varepsilon}\cdot\mathbf{k}}{qk}\right]P_l{}^++\left[-iM_l{}^-\boldsymbol{\varepsilon}\cdot\mathbf{l}_k+E^+{}_{l-1}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\varepsilon}\cdot\mathbf{k}\times\mathbf{l}_k}{qk}+iL^+{}_{l-1}\frac{\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\varepsilon}\cdot\mathbf{k}}{qk}\right]P^-{}_l\right\}\cdot(2l+1)P_l(\hat{q}\cdot\hat{k}), \tag{2.13}$$

where

$$P_l{}^+=\frac{l+1+\boldsymbol{\sigma}\cdot\mathbf{l}_q}{2l+1},\quad P_l{}^-=\frac{l-\boldsymbol{\sigma}\cdot\mathbf{l}_q}{2l+1},\quad \mathbf{l}_q=\frac{\mathbf{q}\times\boldsymbol{\nabla}_q}{i},$$

and $P_l(\hat{q}\cdot\hat{k})$ are the Legendre polynomials. The amplitudes $M_l{}^{+,-}$, $E_l{}^{+,-}$, $L_l{}^{+,-}$ refer to magnetic, electric, and longitudinal vector-meson transitions produced with the lth-wave pseudoscalar meson in the total angular momentum $l\pm\frac{1}{2}$ states. Performing the operations of $\mathbf{l}_k$ and $\mathbf{l}_q$ in Eq. (2.13) and comparing to (2.12) we obtain for the F_i

$$F_1=\sum_{l=0}^{\infty}\{(lM_l{}^++E_l{}^+)P_{l+1}{}'(x)+[(l+1)M_l{}^-+E_l{}^-]P_{l-1}{}'(x)\},$$
$$F_2=\sum_{l=1}^{\infty}\{(l+1)M_l{}^++lM_l{}^-\}P_l{}'(x),$$
$$F_3=\sum_{l=1}^{\infty}\{(E_l{}^+-M_l{}^+)P_{l+1}{}''(x)+(E_l{}^-+M_l{}^-)P_{l-1}{}''(x)\}, \tag{2.14}$$
$$F_4=\sum_{l=1}^{\infty}(M_l{}^+-E_l{}^+-M_l{}^--E_l{}^-)P_l{}''(x),$$
$$F_5=-F_1-xF_3+\sum_{l=0}^{\infty}[L_l{}^+P_{l+1}{}'(x)-L_l{}^-P_{l-1}{}'(x)],$$
$$F_6=-xF_4+\sum_{l=1}^{\infty}(L_l{}^--L_l{}^+)P_l{}'(x),$$

where $x=\hat{q}\cdot\hat{k}$. Here the multipole expansion shows explicitly that for a massive vector field the transverse multipoles EJ and MJ do not exist for $J=0$, where J is the total angular momentum of the vector particle, but the longitudinal one LO exists. This has a simple meaning. The state with total zero angular momentum is a spherically symmetric one, but a spherically symmetric vector field can only be a longitudinal one.

The amplitudes F_i defined in Eq. (2.12) are related

[12] R. Stora, University of Maryland Technical Report No. 250 (unpublished). P. Dennery, Phys. Rev. 124, 2000 (1961). Our expressions for F_5 and F_6 differ from Dennery's. We are grateful to W. Dunn for checking the multipole expansions.

to the A_i defined in Eqs. (2.5) through (2.11) as follows:

$$8\pi aWF_1=\left[\frac{2\mathbf{k}\cdot\mathbf{q}}{(E_1+M_1)(E_2+M_2)}-1\right]A_1-\frac{2(W+M_2)\mathbf{k}\cdot\mathbf{q}+k^2(E_1+M_1)+k_0(E_1+M_1)(E_2+M_2)}{(E_1+M_1)(E_2+M_2)}A_4,$$

$$8\pi aWF_2=\frac{qk}{(E_1+M_1)(E_2+M_2)}A_1-\frac{qk(W+M_2)}{(E_1+M_1)(E_2+M_2)}A_4,$$

$$8\pi aWF_3=\frac{-2qk}{(E_1+M_1)(E_2+M_2)}A_1-\frac{qk}{E_2+M_2}A_2+\frac{qk}{E_2+M_2}A_3+\frac{2qk(W+M_2)}{(E_1+M_1)(E_2+M_2)}A_4 -\frac{qk(W+M_2)}{E_2+M_2}A_5+\frac{qk(W+M_2)}{E_2+M_2}A_6,$$

$$8\pi aWF_4=\frac{q^2}{E_1+M_1}A_2-\frac{q^2}{E_1+M_1}A_3-\frac{q^2[k^2+k_0(E_2+M_2)]}{(E_1+M_1)(E_2+M_2)}A_5+\frac{q^2[k^2+k_0(E_2+M_2)]}{(E_1+M_1)(E_2+M_2)}A_6, \qquad (2.15)$$

$$8\pi aWF_5=-\frac{k^2}{k_0(E_2+M_2)}A_1-\frac{k^2(W+E_1)}{k_0(E_2+M_2)}A_2-\frac{k^2q_0}{k_0(E_2+M_2)}A_3+\frac{k^2(W+M_2)}{k_0(E_2+M_2)}A_4 -\frac{k^2(W+M_2)(W+E_1)}{k_0(E_2+M_2)}A_5-\frac{k^2q_0(W+M_2)}{k_0(E_2+M_2)}A_6,$$

$$8\pi aWF_6=-\frac{qk(W+M_2)}{k_0(E_1+M_1)(E_2+M_2)}A_1+\frac{qk(W+E_1)}{k_0(E_1+M_1)}A_2+\frac{qkq_0}{k_0(E_1+M_1)}A_3+\frac{qkM_v^2}{k_0(E_1+M_1)(E_2+M_2)}A_4 -\frac{qk(W+E_1)[k^2+k_0(E_2+M_2)]}{k_0(E_1+M_1)(E_2+M_2)}A_5-\frac{qkq_0[k^2+k_0(E_2+M_2)]}{k_0(E_1+M_1)(E_2+M_2)}A_6,$$

where $a=[(E_1+M_1)(E_2+M_2)]^{-1/2}$.

In terms of F_i, the differential cross section for the production of an ω meson with polarization vector $\boldsymbol{\varepsilon}$ is obtained to be

$$\begin{aligned} d\sigma/d\Omega=(k/q)\{&[|F_1|^2-2\,\mathrm{Re}(F_1{}^*F_2)\cos\theta]\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}+|F_2|^2[(\mathbf{k}\times\boldsymbol{\varepsilon})^2/k^2] \\ &+[|F_3|^2+|F_4|^2+2\,\mathrm{Re}(F_1{}^*F_4+F_2{}^*F_3)+2\,\mathrm{Re}F_3{}^*F_4\cos\theta][(\mathbf{q}\cdot\boldsymbol{\varepsilon})^2/q^2] \\ &+[|F_5|^2+|F_6|^2+2\,\mathrm{Re}(F_1{}^*F_5)-2\,\mathrm{Re}(F_2{}^*F_5-F_5{}^*F_6)\cos\theta][(\mathbf{k}\cdot\boldsymbol{\varepsilon})^2/k^2] \\ &+2\,\mathrm{Re}[F_1{}^*F_3+F_1{}^*F_6+F_3{}^*F_5+F_4{}^*F_6+F_1{}^*F_2+F_2{}^*F_5 \\ &+(F_3{}^*F_6-F_2{}^*F_3+F_4{}^*F_5)\cos\theta](\mathbf{k}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\boldsymbol{\varepsilon}/kq)\}. \end{aligned} \qquad (2.16)$$

Summing over the polarization of the ω meson by using

$$\sum_{\lambda=1}^{3}\epsilon_i{}^{(\lambda)}\epsilon_j{}^{(\lambda)}=\delta_{ij}+(k_ik_j/M_v{}^2),$$

one arrives at

$$\begin{aligned} d\sigma/d\Omega=(k/q)\{&(3+k^2/M_v{}^2)|F_1|^2+2|F_2|^2+(1+k^2\cos^2\theta/M_v{}^2)(|F_3|^2+|F_4|^2)+(1+k^2/M_v{}^2)(|F_5|^2+|F_6|^2) \\ &-4\,\mathrm{Re}(F_1{}^*F_2)\cos\theta+2\,\mathrm{Re}(F_2{}^*F_3)\sin^2\theta+2(1+k^2\cos^2\theta/M_v{}^2)\,\mathrm{Re}(F_1{}^*F_4)+2(1+k^2\cos^2\theta/M_v{}^2) \\ &\times\mathrm{Re}(F_3{}^*F_4)\cos\theta+2(1+(k^2/M_v{}^2))\,\mathrm{Re}(F_1{}^*F_5)+2(1+(k^2/M_v{}^2))\,\mathrm{Re}(F_3{}^*F_6+F_4{}^*F_5)\cos^2\theta \\ &+2(1+(k^2/M_v{}^2))\,\mathrm{Re}(F_1{}^*F_6+F_3{}^*F_5+F_4{}^*F_6+F_5{}^*F_6+F_1{}^*F_3)\cos\theta\}. \end{aligned} \qquad (2.17)$$

The polarization P_n of the produced baryon along $\hat{n}=\mathbf{q}\times\mathbf{k}/|\mathbf{q}\times\mathbf{k}|$ is (with unpolarized target):

$$\begin{aligned} (d\sigma/d\Omega)P_n=2(k/q)\,\mathrm{Im}\{&F_1{}^*F_2\sin\theta\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}+[F_1{}^*F_3-F_2{}^*F_4+(F_1{}^*F_4-F_2{}^*F_3)\cos\theta-F_3{}^*F_4\sin^2\theta](k(\mathbf{q}\cdot\boldsymbol{\varepsilon})^2/q|\mathbf{q}\times\mathbf{k}|) \\ &+[-F_1{}^*F_4+F_1{}^*F_5-F_2{}^*F_6+(F_1{}^*F_2-F_1{}^*F_3+F_2{}^*F_4-F_2{}^*F_5+F_1{}^*F_6)\cos\theta+F_2{}^*F_3\cos^2\theta \\ &+(F_4{}^*F_5-F_3{}^*F_6)\sin^2\theta][(\mathbf{q}\cdot\boldsymbol{\varepsilon})(\mathbf{k}\cdot\boldsymbol{\varepsilon})/|\mathbf{q}\times\mathbf{k}|]+[-F_1{}^*F_2-F_1{}^*F_6+(F_2{}^*F_6-F_1{}^*F_5)\cos\theta \\ &+F_2{}^*F_5\cos^2\theta-F_5{}^*F_6\sin^2\theta][q(\mathbf{k}\cdot\boldsymbol{\varepsilon})^2/k|\mathbf{q}\times\mathbf{k}|]\}. \end{aligned} \qquad (2.18)$$

For unpolarized ω one has

$$(d\sigma/d\Omega)P_n=2(k/q)\sin\theta\,\mathrm{Im}\{2F_1{}^*F_2+F_1{}^*F_3-F_2{}^*F_4-F_2{}^*F_3\cos\theta-(k^2/M_v{}^2)F_1{}^*F_4\cos\theta$$
$$-F_3{}^*F_4(1+(k^2\cos^2\theta/M_v{}^2))-(F_1{}^*F_6+F_5{}^*F_6)(1+(k^2/M_v{}^2))+(1+(k^2/M_v{}^2))(F_4{}^*F_5-F_3{}^*F_6)\cos\theta\}. \quad (2.19)$$

We envisage our treatment to be applicable to reactions like $\pi^++n\rightarrow p+\omega$,[8] $K^-+p\rightarrow\Lambda+\omega$,[9] $K^-+p\rightarrow\Lambda+\varphi$, etc. In the following sections we shall consider the first reaction in detail, while the work on $K^-+p\rightarrow\Lambda+\omega(\varphi)$ is deferred to a latter communication. Obviously, our calculation would also apply without changes to the isospin reflected reaction $\pi^-+p\rightarrow n+\omega$.

III. MODEL FOR PION PRODUCTION OF ω MESON

As it was stated in Sec. II, the amplitudes A_i defined in (2.5) with (2.6) can be assumed to satisfy a Mandelstam representation. However, we shall not try to use this representation; instead, for the energy region close to threshold of production, we assume the validity of the Cini-Fubini one-dimensional representation for the amplitudes A_i.

$$A_i(u,s,t)=\frac{R_i{}^{(s)}}{s-M_s{}^2}+\frac{R_i{}^{(u)}}{u-M_u{}^2}+\frac{R_i{}^{(t)}}{t-M_t{}^2}+\frac{1}{\pi}\int\frac{\sigma_1(s',t)}{s'-s}ds'+\frac{1}{\pi}\int\frac{\sigma_2(u',t)}{u'-u}du'+\frac{1}{\pi}\int\frac{\sigma_3(t',s)}{t'-t}dt'.$$

Among the various contributions in the three channels, we shall keep only the one-particle and the two-particle lower states, the spectral functions for the latter ones being approximated by the existing resonant states in the respective channel.

Figure 2 shows the singularities in the three channels. In the s channel ($\pi^++n\rightarrow p+\omega$), there is the nucleon pole and the branch cut beginning at the elastic threshold, $(M_\pi+M_N)^2$. In the region between this branch point and the physical threshold for ω production ($s^{1/2}=1720$ MeV), we approximate the π-N cut by the two isospin-$\frac{1}{2}$ π-N resonances, 1517-MeV N^{**} in $D_{3/2}$ state and 1638-MeV N^{***} in $F_{5/2}$ state. The region above the production threshold is approximated by the 2190-MeV $N_1{}^*$, which we tentatively assume from preliminary experimental results to be a $G_{7/2}$ state and which might play an important role in ω production just above 2-BeV center-of-mass total energy.

The u channel has the same singularities as the s channel. As for the π-N resonances, their contributions are expected to be small due to the large distance from the physical region, and hence we shall omit these terms in our calculation.

The t-channel singularities start with a branch cut from the two pions' contribution. The two pions coupled here to ω and π must be in the $T=1$ state. Electromagnetic isovector form factor studies have shown that the ρ meson gives probably the major contribution to this two-pion state. We shall henceforth replace the 2π cut by an effective ρ-meson pole. The next branch cuts begin at the four-pions intermediate state, π-ω state, ρ-ρ state, etc. There is evidence for a resonance in the π-ω state, the B meson[13] of mass $\simeq 1.22$ BeV which can be used to approximate the π-ω cut.

TABLE I. Location of t and u singularities in the $\cos\theta$ plane for the reaction $\pi+N\rightarrow\omega+N$.

$s^{1/2}$ (BeV)	$\cos\theta_{N^{***}}$	$\cos\theta_{N^{**}}$	$\cos\theta_{\pi+N}$	$\cos\theta_N$	$\cos\theta_{2\pi}$	$\cos\theta_\rho$	$\cos\theta_B$
1.74	−19.60	−16.26	−9.09	−7.31	2.73	5.85	11.62
1.82	− 8.34	− 6.98	−4.07	−3.35	1.47	2.74	5.09
1.90	− 6.16	− 5.25		−2.73		2.11	3.75
2.20	− 3.27	− 2.86		−1.70		1.48	2.23

In order to exhibit a clearer picture of the role of the t- and u-channel singularities, we give in Table I their location in the $\cos\theta$ plane for the total energies $s^{1/2}=1.74$, 1.82, 1.90, and 2.20 BeV.

The ρ pole and N pole in the exchange channels are approximately equally distant from the physical region. Hence, for residues of comparable magnitude, one would expect both of them to be important. This throws some light on the question of why a single-particle exchange does not prove to be the dominant mechanism.

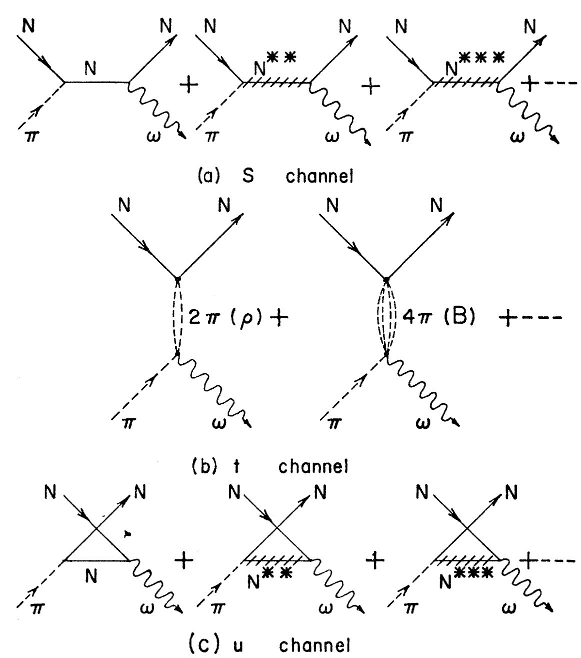

FIG. 2. Cutkosky diagrams: (a) s channel, (b) t channel, (c) u channel of the pion, vector meson, two-nucleon problem.

[13] M. Abolins, R. Lander, W. Mehlhop, N. Xuong, and P. Yager, Phys. Rev. Letters 11, 381 (1963).

The energy range of the experiment of Fields *et al.*[8] extends from threshold (1720 MeV) to approximately 1900-MeV total c.m. energy. This range is obtained in the reaction $\pi^+ + d \to p + p + \omega$, interpreted in the impulse approximation, from the internal motion of nucleons in the deuteron. The most striking features of the experiment are (a) a very fast rising production cross section above threshold, (b) an ω decay distribution indicating that ρ exchange is not the dominant process in this energy range, (c) a forward-to-backward production ratio in the c.m. system close to 2. Walker *et al.*[14] also obtained two points for the total cross section at higher energies (2120 and 2200 MeV), which join quite smoothly with the curve of Ref. 8. We shall try to account for these data by using the following contributions: the nucleon pole and the π-N resonances in the direct channel, and the nucleon exchange and ρ-meson exchange in the crossed channels.

IV. INTERACTION TERMS

In the case of ω-meson production, we shall substitute the following symbols into Sec. II: $M = M_1 = M_2 =$ nucleon mass, $m_\omega = M_V = \omega$-meson mass and $\mu = M_p =$ pion mass.

The contribution of the three pole terms to the amplitudes A_i in (2.5) is found to be

$$A_1{}^0 = -g_{\omega\rho\pi} g_{\rho c} \frac{s-u}{\sqrt{2} m_\omega (t-m_\rho{}^2)},$$

$$A_2{}^0 = g_\pi g_\omega \sqrt{2}\left(\frac{1}{s-M^2} + \frac{1}{u-M^2}\right) - g_{\omega\rho\pi} g_{\rho m} \frac{t+m_\omega{}^2-\mu^2}{\sqrt{2} M m_\omega (t-m_\rho{}^2)},$$

$$A_3{}^0 = g_\pi g_\omega \sqrt{2}\left(\frac{1}{s-M^2} - \frac{1}{u-M^2}\right) - g_{\omega\rho\pi} g_{\rho m} \frac{s-u}{\sqrt{2} m_\omega M (t-m_\rho{}^2)}, \tag{4.1}$$

$$A_4{}^0 = g_\pi g_\omega \sqrt{2}\left(\frac{1}{s-M^2} + \frac{1}{u-M^2}\right) + g_{\omega\rho\pi} g_{\rho c} \frac{4M}{\sqrt{2} m_\omega (t-m_\rho{}^2)} - g_{\omega\rho\pi} g_{\rho m} [\sqrt{2} t / m_\omega M (t-m_\rho{}^2)],$$

$$A_5{}^0 = g_{\omega\rho\pi} g_{\rho c} [\sqrt{2}/m_\omega (t-m_\rho{}^2)],$$

$$A_6{}^0 = 0.$$

The dimensionless coupling constants are defined through the following effective interaction Lagrangian[15]:

$$L_I = g_\pi \bar{N}\gamma_5 \boldsymbol{\tau} N \boldsymbol{\pi} + g_\omega \bar{N}\gamma_\mu N \omega^\mu + g_{\rho c} \bar{N}\gamma_\mu \boldsymbol{\tau} N \boldsymbol{\rho}^\mu + (g_{\rho m}/2M)\bar{N}\sigma_{\mu\nu}\boldsymbol{\tau} N \boldsymbol{\rho}^{\mu\nu} + (g_{\omega\rho\pi}/4m_\omega)\epsilon_{\mu\nu\sigma\lambda}\omega^{\mu\nu}\boldsymbol{\rho}^{\sigma\lambda}\boldsymbol{\pi}, \tag{4.2}$$

with

$$\sigma_{\mu\nu} = 1/2i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu), \quad \rho_{\mu\nu} = \rho_{\mu,\nu} - \rho_{\nu,\mu}, \quad \text{and} \quad \epsilon_{\mu\nu\sigma\lambda}$$

is the fourth-order completely antisymmetric tensor. We have assumed no magnetic coupling for the ω, guided by the knowledge that the isoscalar anomalous magnetic moment of the nucleon is very small. This leads to the result $A_6{}^0 = 0$. For the propagator of the ρ meson we use $D_{\mu\nu} = (g_{\mu\nu} - (p_\mu p_\nu / m_\rho{}^2))/(p^2 - m_\rho{}^2)$, neglecting its width since the ρ pole is rather far away from the physical region.

Combining Eq. (4.1) and Eq. (2.15) we obtain for the pole terms

$$F_1{}^0 = \frac{a}{8\pi W}\left\{ g_\pi g_\omega \frac{\sqrt{2} c (t-m_\omega{}^2-\mu^2)}{(s-M^2)(u-M^2)} + \frac{g_{\omega\rho\pi}}{m_\omega (t-m_\rho{}^2)}\left[\frac{g_{\rho c}}{\sqrt{2}}\left((s-u)\left(\frac{1}{a^2} - 2kqx\right) - 4cM\right) + g_{\rho m}\sqrt{2} c \frac{t}{M}\right]\right\},$$

$$F_2{}^0 = \frac{aqk}{8\pi W}\left\{ g_\pi g_\omega \frac{\sqrt{2}(W+M)(t-m_\omega{}^2-\mu^2)}{(s-M^2)(u-M^2)} - \frac{g_{\omega\rho\pi}}{m_\omega (t-m_\rho{}^2)}\left[\frac{g_{\rho c}}{\sqrt{2}}(s-u+4M(W+M)) - g_{\rho m}\sqrt{2}(W+M)\frac{t}{M}\right]\right\},$$

$$F_3{}^0 = \frac{aqk}{4\pi W}\left\{ g_\pi g_\omega \sqrt{2}\left(\frac{W+M}{s-M^2} + \frac{q_0}{u-M^2}\right) + \frac{g_{\omega\rho\pi}}{\sqrt{2} m_\omega (t-m_\rho{}^2)} \times\left[g_{\rho c}(s-u-(W+M)(E_1-3M)) - g_{\rho m}\frac{(E_1+M)(s-M^2-m_\omega{}^2)+2t(W+M)}{M}\right]\right\},$$

[14] W. D. Walker, E. West, A. R. Erwin, and R. H. March, in *Proceedings of the 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 43. We are grateful to Professor T. Fields for informing us of the corrected results obtained by Walker *et al.*

[15] We have used $\boldsymbol{\tau}/2$ in the definition of $\rho\bar{N}N$ coupling in order to facilitate the comparison with the customary definition and with the theoretical conjecture which equates the $\rho\bar{N}N$ and $\rho\pi\pi$ coupling constants. See, for example, J. J. Sakurai, in Proc. School of Physics "Enrico Fermi," 1962 (to be published).

$$F_4{}^0=\frac{aq^2}{4\pi W}\left\{g_\pi g_\omega\sqrt{2}\frac{E_2+M}{u-M^2}-\frac{g_{\omega\rho\pi}}{\sqrt{2}m_\omega(t-m_\rho{}^2)}\left[g_{\rho c}(k_0W+k_0M-m_\omega{}^2)-g_{\rho m}\frac{(E_2+M)(s-M^2-m_\omega{}^2)}{M}\right]\right\},\qquad(4.3)$$

$$F_5{}^0=\frac{bk^2}{8\pi Wk_0}\left\{g_\pi g_\omega\sqrt{2}\left[\frac{M-W}{s-M^2}+\frac{MW-M^2+\mu^2}{W(u-M^2)}\right]+\frac{g_{\omega\rho\pi}}{m_\omega(t-m_\rho{}^2)}\right.$$
$$\left.\times\left[\frac{g_{\rho c}}{\sqrt{2}}(t-m_\omega{}^2-\mu^2+2(M-E_1)(W+M))-g_{\rho m}\sqrt{2}\frac{Mt-W(m_\omega{}^2-\mu^2)-q_0(s-m_\omega{}^2-M^2)}{M}\right]\right\},$$

$$F_6{}^0=\frac{aqk}{8\pi Wk_0}\left\{g_\pi g_\omega\sqrt{2}\left[\frac{(W+M)^2}{s-M^2}+\frac{2E_1(E_2+M)+m_\omega{}^2}{u-M^2}\right]+\frac{g_{\omega\rho\pi}}{m_\omega(t-m_\rho{}^2)}\left[g_{\rho c}\sqrt{2}(m_\omega{}^2(E_1+M)+kq(W+M)x)\right.\right.$$
$$\left.\left.-\frac{g_{\rho m}}{\sqrt{2}}\frac{(W+E_1)(E_2+M)(m_\omega{}^2-\mu^2+t)+q_0(E_2+M)(s-u)+2m_\omega{}^2t}{M}\right]\right\},$$

where

$$x=\cos\theta,\quad a=[(E_1+M)(E_2+M)]^{-1/2},$$
$$b=[(E_1+M)/(E_2+M)]^{1/2},$$

and

$$c=(E_1+M)[k_0(W+M)-m_\omega{}^2]+2(W+M)kqx.$$

For the amplitudes of π-N resonances we assume a Breit-Wigner formula

$$\mathrm{Im}F=\frac{1}{4W}\frac{\Gamma(\Gamma_i\Gamma_f)^{1/2}}{(W-W_0)^2+(\Gamma^2/4)}.$$

In the case of Γ much smaller than W, and W close to W_0, we find it accurate enough to introduce the resonant amplitudes with a form

$$F=\frac{(\Gamma_i\Gamma_f)^{1/2}}{s_0-s-i\Gamma s_0{}^{1/2}}\qquad(4.4)$$

directly into the multipole amplitudes. The resonances included are[16] the $T=\frac{1}{2}$, $D_{3/2}$ state at $W_1=1517$ MeV with $\Gamma_1=60$ MeV, the $T=\frac{1}{2}$, $F_{5/2}$ state at $W_2=1683$ MeV with $\Gamma_2=80$ MeV, and the $T=\frac{1}{2}$, $G_{7/2}$(?) state at $W_3=2190$ MeV with $\Gamma_3=200$ MeV.

From conservation of angular momentum and parity we see that these resonances contribute to the following multipole amplitudes:

$D_{3/2}$ to $E_2^-(E1, l_\omega=0, 2)$, $L_2^-(L1, l_\omega=0, 2)$, and $M_2^-(M2, l_\omega=2)$;

$F_{5/2}$ to $E_3^-(E2, l_\omega=1, 3)$, $L_3^-(L2, l_\omega=1, 3)$, and $M_3^-(M3, l_\omega=3)$;

$G_{7/2}$ to $E_4^-(E3, l_\omega=2, 4)$, $L_4^-(L3, l_\omega=2, 4)$, and $M_4^-(M4, l_\omega=4)$,

where l_ω is the orbital angular momentum of the ω meson.

The energy dependence of Γ_i and Γ_f will be exhibited as follows: For the initial width Γ_i, as we deal with high energy pions, we can assume practically no change with energy in the range of interest, i.e., Γ_i= a constant. For Γ_f we take $\Gamma_f=\gamma_f k^{2l_\omega}$ in order to exhibit the correct threshold behavior, where γ_f varies slowly with energy. Since, in the energy range not far away from threshold, the particle is predominantly produced in the lower angular momentum state, we shall include only the lower l_ω in the partial width for electric and longitudinal transitions. Then we have

$$N^{**}:\quad E_2^-=\frac{W_1D_E}{s_1-s-iW_1\Gamma_1},\quad L_2^-=\frac{W_1D_L}{s_1-s-iW_1\Gamma_1},\quad M_2^-=\frac{k^2D_M}{W_1(s_1-s-iW_1\Gamma_1)},$$
$$N^{***}:\quad E_3^-=\frac{kF_E}{s_2-s-iW_2\Gamma_2},\quad L_3^-=\frac{kF_L}{s_2-s-iW_2\Gamma_2},\quad M_3^-=\frac{k^3F_M}{s_2(s_2-s-iW_2\Gamma_2)},\qquad(4.5)$$
$$N_1{}^*:\quad E_4^-=\frac{k^2G_E}{W_3(s_3-s-iW_3\Gamma_3)},\quad L_4^-=\frac{k^2G_L}{W_3(s_3-s-iW_3\Gamma_3)},\quad M_4^-=\frac{k^4G_M}{s_3W_3(s_3-s-iW_3\Gamma_3)},$$

[16] M. Roos, Rev. Mod. Phys. **35**, 314 (1963).

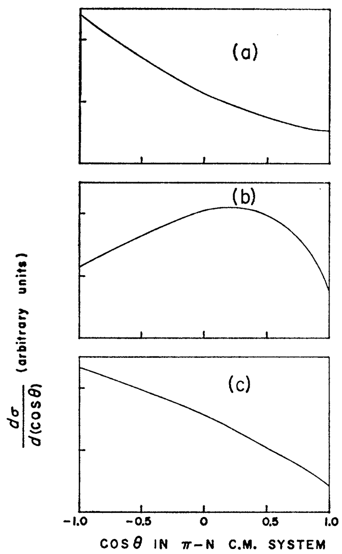

FIG. 3. Separate calculated contributions at $W=1780$ MeV to the differential cross section of the process $\pi+n\rightarrow\omega+p$ from (a) nucleon pole, (b) ρ-charge coupling ($g_{\rho c}$), (c) ρ-magnetic coupling ($g_{\rho m}$).

where $s_i=W_i^2$, $i=1, 2, 3$. The dimensionless constants D_E, D_L, etc., are related by Eqs. (4.4) and (4.5) to the partial widths.

Finally, the total amplitudes are expressed as the sum of pole and multipole amplitudes,

$$F_i=F_i^0+F_i^1, \tag{4.6}$$

where F_i^0 are given in Eq. (4.3) and F_i^1 are obtained from Eq. (2.14) by keeping only E_l^-, M_l^-, L_l^- with $l=2, 3, 4$ in the summation.

V. NUMERICAL RESULTS

In this section we give the results of a "first-step" calculation including only pole terms, i.e., the nucleon pole in the s and u channels and ρ exchange in the t channel with the amplitudes given in Eq. (4.3). A complete numerical analysis of cross sections and polarizations will be presented in a subsequent article.

Out of the five renormalized coupling constants entering into our expressions, two are fairly well known and we take them as $g_\pi^2/4\pi=14.4$ and $g_{\omega\rho\pi}^2/4\pi=10.5$. This $\omega\rho\pi$ coupling constant gives a partial decay width for $\omega\rightarrow(\rho\pi)\rightarrow 3\pi$ of about 8 MeV.[17] Therefore, we have three parameters at our disposal, g_ω, $g_{\rho c}$, and $g_{\rho m}$.

In Fig. 3 we give the separate calculated contributions of ρ exchange (charge coupling and magnetic coupling) and the nucleon pole (direct plus crossed) to the differential cross sections. These curves are given for $W=1780$ MeV (see Fig. 6 for $W=1820$ MeV). The experimental differential cross section (Fig. 4) from Ref. 8 includes 295ω events in the energy range from $W=1710$ to 1890 MeV with most of the events approximately equally distributed between 1740 and 1860 MeV. It is obvious that none of the theoretical curves in Fig. 3 alone has the experimental behavior.

The total cross section is given in Fig. 5. The open circles are from the experiment of Ref. 8 which is our main source of experimental information. The two solid circles are from Ref. 14. The last two points of Ref. 8 around 1870 MeV should not be considered too seriously because here the impulse approximation becomes less reliable. We also calculated the energy dependence of the individual contributions to the total cross section in the low-energy region (<1860 MeV). Comparing with the experimental data in Fig. 5, we find that the $g_{\rho c}$ term gives too fast an increase of the total cross section. The ρ magnetic term and the nucleon term which give entirely wrong differential cross sections show a very slow increase. It is evident that no single process can fit the data.

Including all the terms in Eq. (4.3) we search for the values of the coupling constants which will give the minimum deviations of the calculated differential and total cross sections from the experimental data. At this preliminary stage the following set already gives a rather satisfactory fit.

$$g_\omega^2/4\pi=1.0\,,\quad g_{\rho c}^2/4\pi=0.4\,,$$
$$g_{\rho m}^2/4\pi=0.4\,,\quad g_{\rho m}/g_{\rho c}=-1.0\,.$$

The sign of $g_\pi g_\omega g_{\omega\rho\pi}g_{\rho m}$ is negative, and we used the predetermined values,

$$g_\pi^2/4\pi=14.4\,,\quad g_{\omega\rho\pi}^2/4\pi=10.5\,.$$

The theoretical curve for the total cross section is given in Fig. 5 while the differential cross sections at

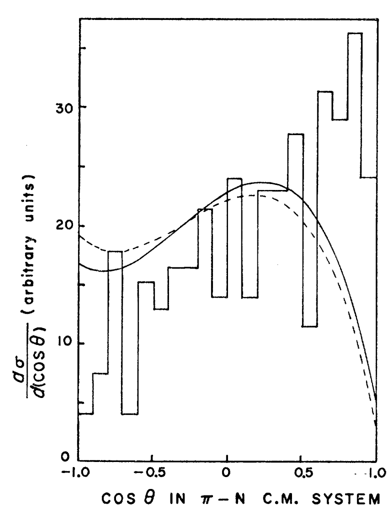

FIG. 4. Differential cross section for $\pi+n\rightarrow\omega+p$. The experimental histogram is taken from Ref. 8 and includes events from the energy range $W=1720$–1890 MeV. The calculated curves are for $W=1780$ MeV (solid line) and $W=1820$ (dotted line), and are normalized to the number of events given in the histogram.

[17] The total decay width of ω is given experimentally as $\Gamma_\omega=9.5\pm2.1$ MeV by N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, J. Steinberger, T. H. Tan, L. Kirsch, and R. Plano, Phys. Rev. Letters **11**, 436 (1963). The ratio of neutral to charged decays of ω is known to be 10–15%.

$W=1780$ and 1820 MeV calculated with these values for the coupling constants are given in Fig. 4.

VI. DISCUSSION

In this work we have presented a general treatment of the vector-meson production in pseudoscalar meson-baryon collisions. A calculation of ω production is made by using a model which is based on the assumptions that the behavior of the production amplitude in the low-energy region is given to a good approximation by including only the closest singularities, and that π-nucleon resonances dominate the dispersion integrals.

In the previous section we have given the results of a numerical calculation in which only the pole terms have been taken into account. Although unitarity is obviously not fulfilled in such an approximation, it still could give sensible results in the threshold region. Indeed, we are able to obtain several significant conclusions.

A single-exchange process does not give an adequate picture to the production amplitude behavior. The influence of ρ exchange and N pole on the production process is shown in Sec. V (see also Fig. 3). The experimental differential cross section indicates that ρ exchange is one of the main mechanisms. However, the crossed nucleon pole is also necessary to give a balanced and reasonably fit picture. The importance of the crossed nucleon pole has already been exhibited in the Chew-Low theory.[18]

With all the pole terms we obtain a good fit to the total cross section up to approximately 1850 MeV,

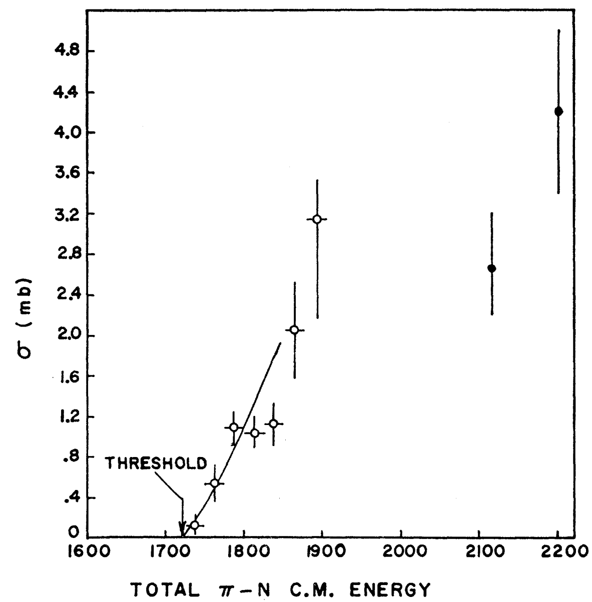

FIG. 5. Experimental and calculated total cross section for $\pi+n\rightarrow\omega+p$. Open circles are from the experiment of Fields *et al.* (Ref. 8) and the two solid circles represent the data of Walker *et al.* (Ref. 14).

[18] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1571 and 1579 (1956).

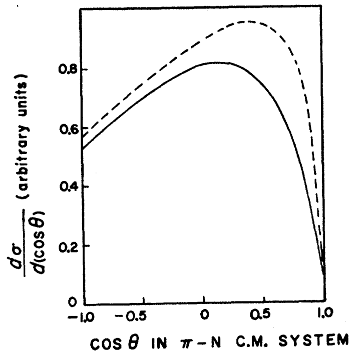

FIG. 6. Calculated differential cross section at $W=1820$ MeV with the $g_{\rho c}$ coupling only. Solid line includes all the amplitudes, while in the dotted line the longitudinal components are omitted.

though only a fair fit to the differential cross section. We remark that the experimental forward-to-backward ratio F/B is about 1.9, while our curves (Fig. 4) give $F/B=1.4$ and 1.25 at 1780 and 1820 MeV, respectively.[18a] The main trouble comes from the dip in the forward direction which is expected to rise up when the π-nucleon resonances are taken into consideration. Only small simultaneous changes to all the coupling constants can be made such that the calculated total cross section will not deviate too much from the experimental data. Even if we relax this requirement we still cannot raise the dip in the forward direction.

From the isovector nucleon electromagnetic form factor analysis, the ratio $g_{\rho m}/g_{\rho c}$ is approximately -1.6 to -2.7.[19] In our fit to the total and differential cross sections, we need $g_{\rho m}/g_{\rho c}$ negative and the value of -1.0 that we obtain is gratifyingly close to the one from form factor calculations.

The trend of the values we obtained agrees with the results of other calculations that the ω-meson coupling is stronger than the ρ-meson coupling to the nucleons. As to their magnitude, our coupling constants squared are about three times smaller than those of Scotti and Wong.[2] On the other side, Beder[5] has recently analyzed π^0 photoproduction above 800-MeV laboratory photon energy and has showed that the contribution of the vector-meson exchange is essential to the description of the recoil nucleon polarization and the higher energy cross sections. His fitting of the data gives

$$(g_\omega^2/4\pi)\Gamma_\omega(\pi^0\gamma)\simeq 1.6\ \text{MeV}.$$

As $\Gamma_\omega(\pi^0\gamma)$ is known to be approximately 1 to 1.5 MeV, this gives a value for $g_\omega^2/4\pi$ in very close agreement to ours.

It is also of interest to mention that if $g_{\omega\rho\pi}$ would be much smaller than its present value (as it was believed

[18a] *Note added in proof*. There is a slight error in the curves of Fig. 4, the correct ones being closer to the experimental points in the region $-1<\cos\theta<-0.25$. This explains the ratios $F/B=1.4$ and 1.25 quoted in the text, which might not be apparent from the drawn curves.

[19] See, for example, M. M. Islam and T. W. Preist, Phys. Rev. Letters **11**, 444 (1963). This paper contains further references. There is a difference by a factor of $\frac{1}{2}$ between our definition of $g_{\rho m}$ and the one used in this article.

before the measurement of the ω width), our fits to the differential cross section would be much poorer.

As to the value of $g_{\rho c}^2/4\pi$ we obtained, we should like ot mention that if the ρ meson is coupled to the isospin current,[20] the universality of the couplings requires $g_{\rho c}^2/4\pi = g_{\rho\pi\pi}^2/4\pi \simeq 0.5$. The value we obtained for $g_{\rho c}^2/4\pi$ in this preliminary numerical work is very close to this number.

The search for the values of the coupling constants was done under the restriction that they should be within one order of magnitude from the "expected" values for a strongly interacting vector meson. We did not search for a fit with values very remote from this region.

If we assume that the ω meson is a pure member of the unitary octet, it is interesting to find from our values for g_ω and $g_{\rho c}$ the mixture of d and f couplings of vector mesons to baryons in the unitary symmetry model. If we let dg be the coefficient of the D-type coupling and fg that of the F-type coupling,[21] we have

$$g_{\rho c} = (d+f)g, \quad g_\omega = (-g/\sqrt{3})(d-3f).$$

Our analysis gives $g_\omega^2/g_{\rho c}^2 = 2.5$. With $d+f=1$ we have two possible sets:

$$\begin{cases} f=0.94 \\ d=-0.06, \end{cases} \quad g_\omega/g_{\rho c} = 1.58;$$

$$\begin{cases} f=-0.44 \\ d=1.44, \end{cases} \quad g_\omega/g_{\rho c} = -1.58.$$

The first set is consistent with pure f coupling while for the second one the d coupling is dominant. For the pseudoscalar meson couplings to baryons the results from various analyses show that d and f (similarly defined) are both positive, with $d/f \simeq 2$–3.

It is, however, more probable that both ω and φ mesons are mixtures of a unitary symmetry singlet (χ_1) with $T=0$ member of an octet (χ_8). If we use for the physical particles the wave functions[22]

$$\omega = (\sqrt{2}/\sqrt{3})\chi_1 - (1/\sqrt{3})\chi_8; \quad \varphi = (1/\sqrt{3})\chi_1 + (\sqrt{2}/\sqrt{3})\chi_8,$$

we have to replace the ω coupling by $g_\omega = g(d-3f)/3 + \sqrt{2}g_1/\sqrt{3}$, where g_1 is the coupling constant of the unitary singlet to the baryon current. In order to obtain the f/d ratio in this case, we need more information. The experimental evidence[23] seems to indicate that the φ coupling to nucleons is much weaker than the ω-coupling. If we assume $g_\varphi = -\sqrt{2}g(d-3f)/3 + g_1/\sqrt{3} \simeq 0$, then one has $g_\omega \simeq (d-3f)g$. Using again the value given by our analysis $(g_\omega^2/g_{\rho c}^2) = 2.5$, we obtain the two sets

$$\begin{cases} f=-0.15 \\ d=1.15 \end{cases} \quad g_\omega/g_{\rho c} = 1.58;$$

$$\begin{cases} f=-0.65 \\ d=0.35 \end{cases} \quad g_\omega/g_{\rho c} = -1.58.$$

Now the first set gives f/d negative and d coupling favored over f coupling. It is interesting that similar conclusions have been obtained[24] from an analysis of nucleon electromagnetic form factors.

Another interesting point is the effect of the F_5 and F_6 amplitudes, which are characteristic of the massive vector meson. Without the inclusion of these terms ("photon-like" behavior) the ρ-exchange term gives a much more pronounced forward contribution, as is shown in Fig. 6. The other terms do not show such a marked change in the differential cross section. The contribution of the F_5, F_6 amplitudes to the total cross section is fairly small in the region close to threshold. For example, at $W=1820$ MeV, the total cross section including all the six F_i amplitudes is 1.46 mb, while it is 1.58 mb without F_5 and F_6 contributions, which is a 8% effect. We may say that the ω meson is produced with a photon-like behavior or transversally at first, and as energy goes higher the longitudinal component is excited with an increasing strength. However, it will be suppressed when the energy is much larger than the rest mass. At present we do not have enough experimental results to study its polarization or alignment.

In concluding, we should like to stress the importance of additional and more detailed experiments of the type meson+baryon → vector meson+baryon, which allow a theoretical analysis and consequently the information on the values of the fundamental coupling constants will follow.

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[20] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954); J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[21] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[22] F. Gürsey, T. D. Lee, and M. Nauenberg (to be published). The wave functions obtained from the model discussed by these authors agree with the empirical mixing angle given, for example, by J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

[23] Y. Y. Lee, W. D. C. Moebs, Jr., B. P. Rose, D. Sinclair, and J. C. Van der Velde, Phys. Rev. Letters **11**, 508 (1963).

[24] W. Alles and S. Bergia, Nuovo Cimento **31**, 262 (1964). These authors also assume that ω and φ are mixed states.